

A competition between T=1 and T=0 pairing in pf -shell nuclei with $N = Z$

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Pairing gain energies of $J=0^+$, $T=1$ and $J=1^+$, $T=0$ states for the $l=3$ and $l=1$ configurations are calculated in the $(1f2p)$ shell model space with $T=1$ and $T=0$ pairing interactions, respectively. It is pointed out that two-body matrix element of the spin-triplet $T=0$ pairing is weakened substantially when the spin-orbit splitting is large in the $1f$ orbits, even the pairing strength is much larger than the spin-singlet $T=1$ pairing interaction. However, the spin-triplet pairing correlations may overcome the spin-singlet pairing correlations in the $2p$ configuration if the spin-triplet pairing strength is more than 50% larger than the spin-singlet pairing. It is also pointed out in the Hartree-Fock wave functions that the mismatching of proton and neutron radial wave functions is at most a few % level, even the Fermi energies are largely different in the proton and neutron potentials. These results imply that the configuration with $J=0^+$, $T=1$ is very likely in the ground states of odd-odd pf shell nuclei even under the influence of the strong spin-triplet $T=0$ pairing, except at the middle of pf shell in which odd proton and odd neutron may occupy the $2p$ orbits. These results are consistent with the observed spin-parity $J^\pi = 0^+$ of all odd-odd pf shell nuclei except $^{70}_{35}\text{Cu}$ which has $J^\pi = 1^+$.

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I. INTRODUCTION

It has been discussed for a long time the role of the neutron-proton isoscalar spin-triplet ($T=0, S=1$) pairing interaction in finite nuclear system [1, 2] since it is stronger than the isovector spin-singlet ($T=1, S=0$) pairing interaction in the nuclear matter [3, 4].

Nevertheless, the nuclei observed favor the spin-singlet $T=1$ pairing between identical particles. A straightforward answer to this puzzle is that most of stable nuclei have different numbers of neutrons and protons, and the $T=0$ pair is hardly made since the proton and neutron occupy the different single-particle orbits near the Fermi surface. Even in nuclei with the equal numbers of protons and neutrons, the $J=1, T=0$ neutron-proton pairing is not a favorable correlation compared with the $J=0, T=1$ pairing as was seen in the ground state spins of odd-odd nuclei in the mass region above $A \geq 20$ [2]. It has been suggested that the nuclear spin-orbit field will suppress largely the spin-triplet pairing than the spin-singlet pairing [5–7]. While so far no clear evidence was found to show the role of $T=0$ pairing in the nuclear ground state, the manifestation of the spin-triplet pairing was discussed in the high-spin states [8, 9] and also in the Gamow-Teller Giant resonances in $N=Z$ nuclei [10].

We study in this paper the quenching of two-body matrix element of $T=0$ pairing interaction in jj coupling scheme in comparison with that of $T=1$ pairing interaction. Its consequence on the gain energies is also discussed for the $J^\pi=0^+$ and the $J^\pi=1^+$ states in the $(1f2p)$ shell model configurations by using the HF single-particle wave functions. The Coulomb interaction is taken into account properly in the HF potential. This paper is orga-

nized as follows. We study the two-body matrix elements of $T=0$ and $T=1$ pairing interactions and the overlap of neutron and proton HF single-particle states in $(1f2p)$ shell model configuration in Section 2. The competition between the gain energies of $T=0$ and $T=1$ pairing interactions is studied by diagonalizing the shell model space for $1f$ and $1p$ configurations in Section 3. A summary is given in Section 4.

II. T=0 AND T=1 TWO-BODY PAIRING INTERACTION

We adopt a separable form of the pairing interaction in this paper. The spin-singlet $T=1$ pairing interaction is defined as a separable form,

$$V^{T=1}(\mathbf{r}, \mathbf{r}') = -G^{T=1} \sum_{i,j} P_{i,i}^{(1,0)\dagger}(\mathbf{r}, \mathbf{r}') P_{j,j}^{(1,0)}(\mathbf{r}, \mathbf{r}') \quad (1)$$

where the pair field operator reads

$$P_{a,b}^{(T,S)\dagger}(\mathbf{r}, \mathbf{r}') = [a_a^\dagger a_b^\dagger]^{(T,S)} \psi_a(\mathbf{r}) \psi_b(\mathbf{r}'). \quad (2)$$

with the single-particle wave function $\psi(\mathbf{r})$. The pairing strength $G^{T=1}$ is fitted to the empirical pairing gaps [4, 6, 11] and given by

$$G^{T=1} = \frac{24}{A}. \quad (3)$$

The absolute value of pairing strength (3) should not be taken seriously since it depends on the model space adopted. However, this value might be a reasonable

choice for the one major shell model space calculations [4, 6, 11].

The spin-triplet $T=0$ pairing is also given by a separable form,

$$V^{T=0}(\mathbf{r}, \mathbf{r}') = -f G^{T=1} \sum_{i \geq i', j \geq j'} P_{i,i'}^{(0,1)\dagger}(\mathbf{r}, \mathbf{r}') P_{j,j'}^{(0,1)}(\mathbf{r}, \mathbf{r}') \quad (4)$$

where f is varied from 1~2 for the strength of $T=0$ pairing interaction. It should be noticed that, for $T=0$ pairing, the pair configurations will be not only in the same orbit with $(l_i = l_{i'}, j_i = j_{i'})$ but also in the spin-orbit partner orbits with $(l_i = l_{i'}, j_i = j_{i' \pm 1})$. The two-body matrix element for the $T=1$ pairing is evaluated to be

$$\begin{aligned} & \langle (j_i j_i) T = 1, J = 0 | V^{(T=1)} | (j_j j_j) T = 1, J = 0 \rangle \\ & = -\sqrt{(j_i + 1/2)(j_j + 1/2)} G^{T=1} I_{ij}^2 \end{aligned} \quad (5)$$

where I_{ij} is the overlap integral,

$$I_{ij} = \int \psi_i(\mathbf{r})^* \psi_j(\mathbf{r}) d\mathbf{r} \quad (6)$$

with the HF single-particle wave function $\psi(\mathbf{r})_i$. For the $T=0$ pairing, the two-body matrix element involves the transformation coefficient of (jj) coupling scheme to (ls) coupling scheme and reads

$$\begin{aligned} & \langle (j_1 j_2) T = 0, J = 1 | V^{(T=1)} | (j'_1 j'_2) T = 0, J = 1 \rangle = \\ & - \langle [(l_1 \frac{1}{2})^{j_1} (l_2 \frac{1}{2})^{j_2}]^{J=1} | [(l_1 l_2)^{L=0} (\frac{1}{2} \frac{1}{2})^{S=1}]^{J=1} \rangle \\ & \langle [(l'_1 \frac{1}{2})^{j'_1} (l'_2 \frac{1}{2})^{j'_2}]^{J=1} | [(l'_1 l'_2)^{L=0} (\frac{1}{2} \frac{1}{2})^{S=1}]^{J=1} \rangle \\ & \times \frac{\sqrt{2l_1+1} \sqrt{2l'_1+1}}{\sqrt{1+\delta_{j_1,j_2}} \sqrt{1+\delta_{j'_1,j'_2}}} f G^{T=1} (I_{j_1 j'_1} I_{j_2 j'_2} + I_{j_1 j'_2} I_{j_2 j'_1}) \end{aligned} \quad (7)$$

where $\langle [(l_1 \frac{1}{2})^{j_1} (l_2 \frac{1}{2})^{j_2}]^{J=1} | [(l_1 l_2)^{L=0} (\frac{1}{2} \frac{1}{2})^{S=1}]^{J=1} \rangle$ is the transformation coefficient and the overlap integral I_{ij} will involve both the proton and neutron wave functions. The transformation coefficient can be given by 9J symbol and the explicit form is tabulated in Table I. The square of the transformation coefficient is 1/6 and 1/3 for $j_1 = j_2$ and $j_1 = j_2 \pm 1$ configurations, respectively, in the limit of large angular momentum $l \rightarrow \infty$. These values suggest large quenchings of the spin-triplet pairing correlations and the spin-orbit partner may contribute largely to the spin-triplet pairing matrix. While in the small l limit, $l \rightarrow 0$, the coefficient is unity for $j = j' = l + 1/2$, and the coefficients are zero for the other 3 configurations. This suggests that the spin-triplet pairing could be large as well as the spin-singlet pairing for the pair configuration in the $s_{1/2}$ orbit, and still substantially large for the configuration in the $p_{3/2}$ orbit.

The overlap integral I_{ij} for the neutron-proton pair is performed using HF wave functions obtained with a Skyrme interaction SLy4. The single-particle energies of

TABLE I: The transformation between jj coupling to ls coupling for the pair wave functions, $R = \langle [(l_1 \frac{1}{2})^{j_1} (l_2 \frac{1}{2})^{j_2}]^{J=1} | [(l_1 l_2)^{L=0} (\frac{1}{2} \frac{1}{2})^{S=1}]^{J=1} \rangle$. Ω denotes a value $\Omega = 3(2l+1)^2$.

j	j'	R	$l = 1$	$l = 3$
$l + 1/2$	$l + 1/2$	$\sqrt{\frac{(2l+2)(2l+3)}{2\Omega}}$	$\frac{1}{3} \sqrt{\frac{10}{3}}$	$\frac{2\sqrt{3}}{7}$
$l + 1/2$	$l - 1/2$	$-\sqrt{\frac{4l(l+1)}{\Omega}}$	$-\frac{2}{3} \sqrt{\frac{2}{3}}$	$-\frac{4}{7}$
$l - 1/2$	$l - 1/2$	$-\sqrt{\frac{2l(2l-1)}{2\Omega}}$	$-\frac{1}{3} \sqrt{\frac{1}{3}}$	$-\frac{\sqrt{5}}{7}$
$l - 1/2$	$l + 1/2$	$\sqrt{\frac{4l(l+1)}{\Omega}}$	$\frac{2}{3} \sqrt{\frac{2}{3}}$	$\frac{4}{7}$

TABLE II: Overlap integrals of proton and neutron HF wave functions in ^{48}Cr and ^{56}Ni and ^{64}Ge . The values are given in unit of %. HF calculations are performed with SLy4 interaction.

ν	π	^{48}Cr	^{56}Ni	^{64}Ge
1f _{7/2}	1f _{7/2}	99.9	100.	99.9
1f _{7/2}	1f _{5/2}	97.7	98.9	99.1
1f _{5/2}	1f _{7/2}	99.4	99.7	99.8
1f _{5/2}	1f _{5/2}	99.6	99.8	99.9
2p _{3/2}	2p _{3/2}	99.6	99.7	99.7
2p _{3/2}	2p _{1/2}	98.2	99.1	98.9
2p _{1/2}	2p _{3/2}	99.8	99.6	99.9
2p _{1/2}	2p _{1/2}	99.1	99.6	99.6

^{56}Ni are shown in Fig. 1 for both neutrons and protons. As is seen in Fig. 1, the Fermi energies of proton and neutron potentials are largely different by about 9 MeV. Nevertheless the overlap integral of proton and neutron wave functions involved in two-body matrix element have rather similar radial shapes and the overlap integrals I_{ij} are close to 1.0, deviating at most 3% as is given in Table II. Thus the quenching due to the mismatching of proton and neutron wave functions in the spin-triplet pairing matrix is rather small compared with that due to the transformation coefficient from (jj) to (LS) couplings. Because of this reason, we neglect the mismatching effect of radial wave functions and the overlap integral is taken to be 1 hereafter. The overlap integral of the pair wave functions will appear also in the case of short-range δ -type neutron-proton pairing interaction in which four radial wave functions are involved in the integral.

III. PAIRING GAIN ENERGY IN pf SHELL CONFIGURATIONS

In Fig.2, the pairing gain energies for the $J^\pi = 0^+$ state with the isospin $T=1$ and the $J^\pi = 1^+$ state with the isospin $T=0$ are plotted as a function of the strength parameter f to the $T=0$ pairing interaction taking the p -shell ($l = 1$) and the f -shell ($l = 3$) configurations. We diagonalize separately the p - and f -shell configurations to disentangle the role of the pairing and the spin-

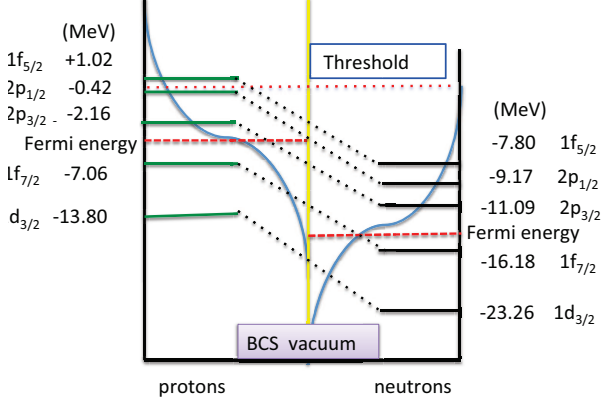


FIG. 1: HF energies of proton and neutron orbits in ^{56}Ni .

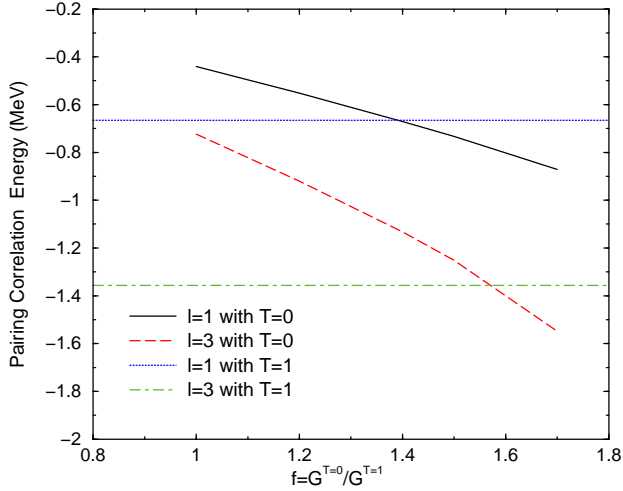


FIG. 2: Pairing gain energies of the lowest ($J=0^+$, $T=1$) and ($J=1^+$, $T=0$) states for the $l=3$ and $l=1$ configurations. The spin-singlet $T=1$ pairing interaction is fixed to be $G^{T=1}=24/A$ MeV with a mass $A=56$, while the spin-triplet $T=0$ pairing is varied with the factor $f=1 \sim 1.7$ multiplied to the $G^{T=1}$ value.

orbit interactions in a transparent way. For the $l=1$ case, the $(2p_{3/2})^2$ and $(2p_{1/2})^2$ configurations are available for the $J^\pi=0^+$ state, while the $(2p_{3/2}2p_{1/2})$ configuration is also available for the $J^\pi=1^+$ state. In a similar way, the $(1f_{7/2})$ and $(1f_{5/2}^2)$ configurations participate to the $J^\pi=0^+$ state in the $l=3$ case, and the $(1f_{7/2}1f_{5/2})$ configuration is also involved in the $J^\pi=1^+$

state. The spin-orbit splitting is parametrized as

$$\Delta\varepsilon_{ls} = -V_{ls}(\mathbf{l} \cdot \mathbf{s}), \quad (8)$$

where the coupling strength is taken to be [12]

$$V_{ls} = \frac{24}{A^{2/3}}. \quad (9)$$

This spin-orbit potential reproduces well the empirical spin-orbit splitting $\Delta\varepsilon=7.0\text{MeV}$ between $1f_{7/2}$ and $1f_{5/2}$ states in ^{41}Ca [13]. The uncertainty of this strength (9) might be less than 20% in the sd and pf shell regions even when we adopt other empirical information of the spin-orbit splittings. Using the pairing matrix elements and the spin-orbit splittings, we diagonalize the model space for the $l=1$ and the $l=3$ configurations, respectively, and the results are shown in Fig. 2. The lowest energy state with $J^\pi=0^+$ for the $l=3$ case gains more binding energy than $J^\pi=1^+$ state for the strength factor $f < 1.5$. In the strong $T=0$ pairing case, $f \geq 1.6$, the $J^\pi=1^+$ state obtain more binding energy than the lowest $J^\pi=0^+$ state. These results are largely due to the quenching of $T=0$ pairing matrix element by the transformation coefficient from (jj) coupling to (LS) coupling scheme. This quenching is never happened for the $T=1$ pairing matrix element since the mapping of the two-particle wave function from (jj) coupling to (LS) coupling is simply implemented by a factor $\sqrt{j+1/2}$ in Eq. (5). For the $l=1$ case, the competition between the $J^\pi=0^+$ and the $J^\pi=1^+$ states is shown also in Fig. 2. Because of the smaller spin-orbit splitting in this case, the coupling among available configurations are rather strong and the lowest $J^\pi=1^+$ state gains more binding energy than the $J^\pi=0^+$ state in the case $f \geq 1.4$. These results are consistent with the observed spins of $N=Z$ odd-odd nuclei in the pf shell where all the ground states have the spin-parity $J^\pi=0^+$, except $^{58}_{29}\text{Cu}$. The ground state of $^{58}_{29}\text{Cu}$ has $J^\pi=1^+$ since the odd proton and odd neutron occupy mainly the $2p$ orbits where the spin-orbit splitting is expected to be much smaller than that of $1f$ orbits seen in Fig. 1.

The mass number dependence of the spin-orbit splitting is approximately determined by the Eq. (8). Since the coupling strength and the largest angular momentum in each major shell are proportional to $A^{-2/3}$ and $A^{1/3}$ [12], respectively, the spin-orbit splitting of the largest angular momentum states is roughly proportional to $A^{-1/3}$. On the other hand, the pairing correlation energy might be proportional to $A^{-1/2}$ as is seen in the pairing gap systematics [4, 14]. Thus, the spin-orbit splitting will decrease slower than the pairing correlation energy as a function of the mass number. As a result, it is expected in medium-heavy nuclei with $N=Z > 30$, that the spin-orbit splitting may hinder more effectively the spin-triplet pairing correlations than lighter nuclei with $N=Z < 30$. In reality, the spin-orbit splitting decreases more slowly than the $A^{-1/3}$ dependence; 6.2 MeV for the $l=1$ states in ^{16}O , 5.5 MeV for the $l=2$ states in ^{40}Ca ,

7.0 MeV for the $l = 3$ states in ^{56}Ni and 7.0 MeV for the $l = 4$ states in ^{100}Sn [15, 16].

It is shown that the shell model matrix elements give the strength factor f in Eq. (4) in the range of 1.6-1.7 for both sd shell and pf shell configurations [7, 17, 18]. In ref. [6], the ratio 1.5 is adopted to analyze the spin-triplet pairing correlations in the $N=Z$ nuclei in the shell model calculations. These adopted values f and the results in Fig. 2 suggests that, in the odd-odd $N=Z$ nuclei, the $J^\pi = 1^+$ state could be a favorite configuration in the ground state rather than the $J^\pi = 0^+$ one especially when the $p_{3/2}$ orbit is the main configuration for the valence particles. However the implementation of spin-triplet pair condensation will not be guaranteed immediately by the spin of the ground state and may need careful examination of many-body wave functions obtained by HF-Bogoliubov or large-scale shell model calculations [19].

IV. SUMMARY

In summary, we studied the sin-singlet and the spin-triplet pairing correlations in the pf shell model configurations in the nuclei with the same proton and neutron numbers $N = Z$. It is pointed out that the spin-triplet

pairing matrix element is largely quenched by the projection of the pair wave function from the (jj) coupling scheme to the (LS) coupling scheme. On the other hand, there is no quenching in the spin-singlet interaction since the $J^\pi = 0^+$ pair in the (jj) coupling scheme has the total spin $S = 0$ and the projection does not involve any quenching factor. The mismatching of the proton and neutron radial wave functions due to the large difference of the Fermi energies is also studied by using the HF wave functions. While the difference between the proton and neutron Fermi energies is quite large as much as 9MeV in the $N = Z = 28$ nucleus, the overlap integral I between proton and neutron wave function in the spin-triplet pairing matrix is rather close to one and the deviation is at most 3%. The spin-triplet pairing correlation energy in the $1f$ shell configuration becomes larger than the spin-singlet pairing when the scaling factor f of the spin-triplet pairing is larger than 1.6. On the other hand, for the $2p$ configuration, the spin-triplet pairing correlation becomes dominant even the factor f is around 1.4.

Acknowledgments

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